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Analysis and Reduction of Power Grid Models under Uncertainty

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Abstract

The increased utilization of alternative energy sources requires that evolving power grid systems be designed with inherent robustness towards variability and uncertainty in generation capacity. Developing smart grid designs that are stable in the face of uncertain power generation requires efficient predictive grid models that account for relevant uncertainties. Accordingly, there is a need for effective methods for analysis of uncertain nonlinear grid dynamics, and for model reduction strategies that allow the efficient modeling and optimization of grid designs.

I will describe some of our recent mathematical and algorithmic developments that are aligned with this broad goal. Focusing on dynamical analysis and reduction of ordinary differential equation (ODE) systems, I will outline our use of computational singular perturbation methods, founded on an eigenanalysis framework, including both their traditional application in deterministic ODE systems and their extension to uncertain systems. Specifically, I will discuss our recent work on stochastic eigenanalysis in ODE systems using polynomial chaos methods for the probabilistic representation of uncertain variables. I will illustrate these developments in model ODE systems. I will also explore the use of these stochastic dynamical analysis results for goal-oriented ODE model reduction. I will conclude with a discussion of key challenges in this overall landscape.

Biography

Habib Najm is a Distinguished Member of the Technical Staff at Sandia National Laboratories in Livermore, CA. His research includes the development of numerical algorithms and codes for computation and analysis of chemically reacting flow, and the development of uncertainty quantification techniques with general application in computational science.

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Outline

Motivation – 1

- Increased electric grid power generation with alternative energy sources leads to increased
  - Variability, and
  - Uncertainty

  in grid power input
- Designing smart grid systems that are robust under these conditions requires predictive grid modeling accounting for uncertainty
- Need for effective methods for
  - analysis of uncertain nonlinear grid dynamics
  - model reduction for grid models with uncertainty

  to enable efficient modeling and design optimization in power grid systems

Motivation – 2

- Interest in nonlinear dynamical power grid models
- Governed by differential algebraic equations (DAEs)
  - Generators: Ordinary differential equations (ODEs)
  - Network: Algebraic constraints
- Grid models are complex
  - Large number of governing equations (dimension N)
  - Large number of connections
  - Strong non-linearity – ODEs/DAEs
  - Large range of time scales – stiffness
- Need for analysis and model reduction methods
  - Methods based on dynamical analysis
    - Automated identification of slow manifolds

Uncertainty in Power Grid Models

- Smart grids involve sources of uncertainty
  - Uncertain distributed generation and load
    - Both uncertain and randomly varying
  - Uncertain grid structure (including faults)
- Need for dynamical analysis methods that
  - Can handle uncertainty
  - Both parametric and structural uncertainty
  - Provide model reduction with quantified fidelity
    - *accounting* for uncertainty

Deterministic Nonlinear ODE System Analysis

- Computational Singular Perturbation (CSP) analysis
- Jacobian eigenvalues provide first-order estimates of the time-scales of system dynamics: $\tau_i \sim 1/\lambda_i$
- Jacobian right/left eigenvectors provide first-order estimates of the CSP vectors/covectors that define decoupled fast/slow subspaces
- With chosen thresholds, have M “fast" modes
  - M algebraic constraints define a slow manifold
  - Fast processes constrain the system to the manifold
  - System evolves with slow processes along the manifold
- CSP *time-scale-aware* Importance indices provide means for elimination of “unimportant" network nodes and connections for a selected observable

3D ODE System Example

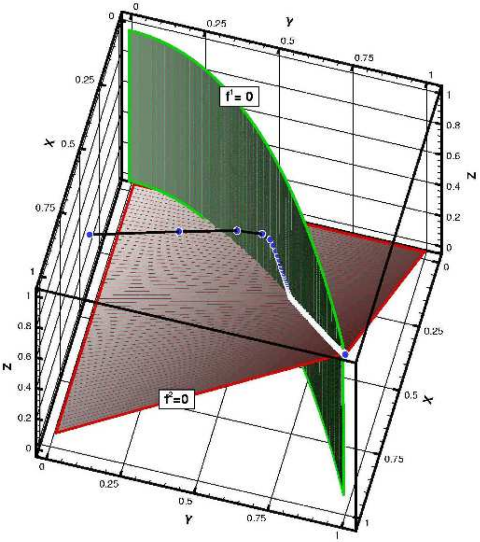
$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\frac{d\mathbf{y}}{dt} = \mathbf{g} = \begin{bmatrix} -\frac{5y_1}{\varepsilon} - \frac{y_1y_2}{\varepsilon} + y_2y_3 + \frac{5y_2^2}{\varepsilon} + \frac{y_3}{\varepsilon} - y_1 \\ 10\frac{y_1}{\varepsilon} - \frac{y_1y_2}{\varepsilon} - y_2y_3 - 10\frac{y_2^2}{\varepsilon} + \frac{y_3}{\varepsilon} + y_1 \\ \frac{y_1y_2}{\varepsilon} - y_2y_3 - \frac{y_3}{\varepsilon} + y_1 \end{bmatrix}$$

$\varepsilon << 1$: small parameter; controls the stiffness of the system

3D ODE System Dynamical Structure

- From any initial condition:
- System cascades through 2D, 1D manifolds to equilibrium



CSP analysis of Power Grid Systems

- ODE system RHS involves sums of contributions by individual generators
- CSP analysis identifies importance of each in the proper dynamical context
- Reduction strategy
  - Select quantities of interest (QoIs)
  - Eliminate generators that have low importance for the QoIs

Analysis of Uncertain ODE Systems

- Handle uncertainties using probability theory
- Every random instance of the uncertain inputs provides a “sample" ODE system
  - Uncertainties in fast subspace lead to uncertainty in manifold geometry
  - Uncertainties in slow subspace lead to uncertain slow time dynamics
- Probabilistic measures of importance
- Probabilistic comparison of models

- One can analyze/reduce each system realization
  - Statistics of $x(t;\lambda)$ trajectories
- This can be expensive!
- Explore alternate means

Polynomial Chaos Methods for UQ

- Model uncertain quantities as random variables (RVs)
- Given a probability space $(\Omega, \mathfrak{S}, P)$
- and any *germ* $\boldsymbol{\xi}(\omega) = \{\xi_1, \cdots, \xi_n\}$ – a set of *i.i.d.* RVs
    - where density of $\boldsymbol{\xi}$ is uniquely determined by its moments
- Any RV in $L^2(\Omega, \mathfrak{S}(\boldsymbol{\xi}), P)$ can be written as a Polynomial Chaos expansion (PCE), thus

$$x(t,\omega) = f(t,\boldsymbol{\xi}) \simeq \sum_{\alpha=0}^{P} x_\alpha(\mathbf{x},t)\Psi_\alpha(\boldsymbol{\xi}(\omega))$$

– $x_\alpha(t)$ are mode strengths
– $\Psi_\alpha()$ are functions orthogonal w.r.t. the density of $\boldsymbol{\xi}$

- with dimension n and order p:

$$P + 1 = \frac{(n+p)!}{n!p!}$$

Essential Use of PC in UQ

Strategy:

- Represent model parameters/solution as random variables
- Construct PCEs for uncertain parameters
- Evaluate PCEs for model outputs

Advantages:

- Computational efficiency
- Sensitivity information

Requirement:

- Random variables in L^2, i.e. with finite variance

Intrusive Galerkin PC UQ: direct/*no-sampling*

- Given model equations: $\mathcal{M}(u(\mathbf{x},t);\lambda) = 0$
- Express uncertain parameters/variables using PCEs

$$u = \sum_{k=0}^{P} u_k \Psi_k; \quad \lambda = \sum_{k=0}^{P} \lambda_k \Psi_k$$

- Substitute in model equations; apply Galerkin projection
- New set of equations: $\mathcal{G}(U(\mathbf{x},t),\Lambda) = 0$
  - with $U = [u_0,\ldots,u_P]^T$, $\Lambda = [\lambda_0,\ldots,\lambda_P]^T$
- Solving this system *once* provides the full specification of uncertain model ouputs

Intrusive Galerkin PC UQ ODE example

$$\frac{du}{dt} = f(u;\lambda)$$

$$\lambda = \sum_{i=0}^{P} \lambda_i \Psi_i \qquad u(t) = \sum_{i=0}^{P} u_i(t)\Psi_i$$

$$\frac{du_i}{dt} = \frac{\langle f(u;\lambda)\Psi_i\rangle}{\langle \Psi_i^2\rangle} \qquad i = 0, \ldots, P$$

Say $f(u;\lambda) = \lambda u$, then

$$\frac{du_i}{dt} = \sum_{p=0}^{P}\sum_{q=0}^{P} \lambda_p u_q C_{pqi}, \quad i = 0, \cdots, P$$

where the tensor $C_{pqi} = \langle \Psi_p \Psi_q \Psi_i \rangle / \langle \Psi_i^2 \rangle$ is readily evaluated

Dynamical Analysis of the Galerkin PC System

Key questions:

- How do the eigenvalues and eigenvectors of the Galerkin system relate to those of the sampled original system
- What can we learn about the sampled dynamics of the original system from analysis of the Galerkin system
  - fast/slow subspaces
  - slow manifolds
- Can CSP analysis of the Galerkin system be used for analysis and reduction of the original uncertain system

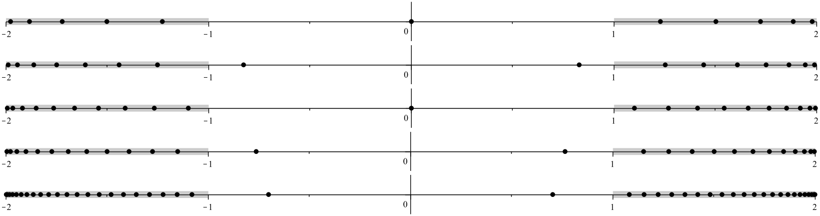
Stochastic system Jacobian: J
Reformulated Galerkin system Jacobian: $\mathcal{J}^P$

Key Results

1. The spectrum of $\mathcal{J}^P$ is contained in the convex hull of the essential range of the random matrix J.
$$\text{spect}(\mathcal{J}^P) \subset \text{conv}(\widetilde{W}(J))$$
2. As $P \to \infty$, the eigenvalues of $\mathcal{J}^P(t)$ converge weakly, *i.e.* in the sense of measures, toward $\bigcup_{\omega\in\Omega} \text{spect}(J(\omega))$.
3. $\mathcal{J}^P$ eigenvalues and eigenpolynomials can be used to construct polynomial approximations of the random eigenvalues and eigenvectors.

Sonday *et al.*, *SISC*, 2011; Berry *et al.*, in review

1D Linear Example



$$\dot{x}(\xi,t) = a(\xi)x(\xi,t); \quad \xi(\omega) \sim U[-1,1];$$

$$J = a(\xi) \equiv \begin{cases} \xi + 1 & \text{for} \quad \xi \geq 0, \\ \xi - 1 & \text{for} \quad \xi < 0. \end{cases}$$

$$\widetilde{W}(J) = [-2,-1] \cup [1,2]; \qquad \text{conv}(\widetilde{W}(J)) = [-2,2].$$

LU PC: eigenvalues of $\mathcal{J}^P$ shown for $P = 10, 15, 20, 25, 45$

A 3D Non-Linear example

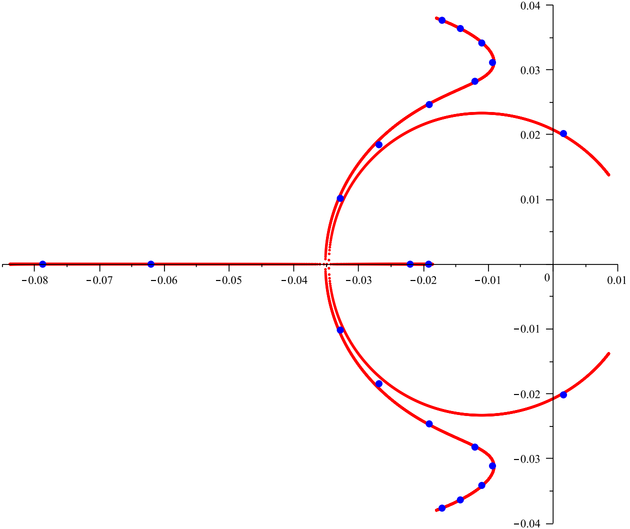
Makeev et al., JCP, 2002

$$\dot{u} = az - cu - 4duv \qquad \dot{v} = 2bz^2 - 4duv$$
$$\dot{w} = ez - fw \qquad z = 1 - u - v - w$$

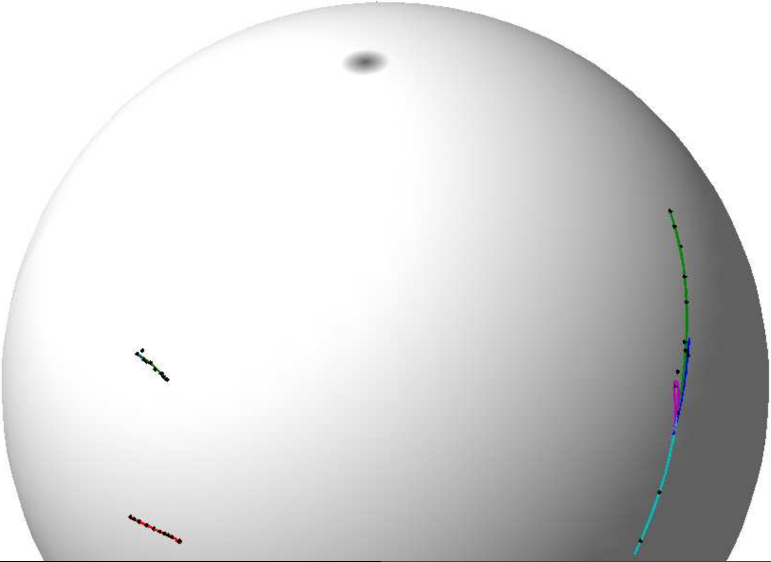
$$a = 1.6,\ b = 20.75 + .45\xi,\ c = 0.04$$
$$d = 1.0,\ e = 0.36,\ f = 0.016$$

$$u(0) = 0.1, v(0) = 0.2, w(0) = 0.7$$

3D Non-Linear Example; PC order 10



3D Non-Linear Example; PC order 10. Eigenvectors.



Closure

- Proposed use of CSP method for analysis of power grid non-linear dynamics
- Probabilistic extension for analysis/reduction of uncertain dynamical systems
- Outlined relationship between eigen-analysis of a sampled stochastic ODE system and the Galerkin PC system.
- Galerkin system eigenvalues/eigenvectors can be used to analyze the dynamics of the stochastic system
- Work in progress on
  - Demonstration of CSP analysis and reduction in sample power grid models
  - Including parametric and structural uncertainty
  - Optimized stochastic model reduction strategies